1) Prove, that $\prod_{k=1}^{N} p_k < 4^{p_N}$, where p_k - k-th prime number

2) Is the set of pairs of natural numbers finite, such that all their prime factors coincide, and the prime factors of the numbers $(a+1)$ and $(b+1)$ also coincide?

3) Do there exist 6 points on a plane (no 3 lying on a single line, the distances between which are measured by distinct integer numbers?

4) Can an irrat. number to an irrat. power be rational?

5) Is it possible to draw an equilat. triangle on squared paper with vertices in the nodes?

6) Is it possible to factor the polynomial $x^{200} \cdot y^{200} + 1$?

7) $a+b+c=1$. Prove that $a^2+b^2+c^2 \geq \frac{1}{3}$

8) $|x|<1$ Find with precision within 0.01 $\left(2-(1-x)^{\frac{1}{2}}\right)^{\frac{1}{3}}$

9) How many digits are in the number 125^{100}?

10) Find $\sin 1°$ with as much precision as possible.

11) $a_1, \dots, a_{n+1}$ form a geom. progression. Find $\sum_{k=1}^{n} \left(\cos a_k \cdot \cos a_{k+1}\right)^{-1}$

12) Find all functions for which for all x_1 and x_2 $\left|f(x_1)-f(x_2)\right| \leq |x_1-x_2|^2$

13) How many solutions does the eq-n $16^{-x} = \log_{\frac{1}{16}} x$ have

14) Solve: $x^{x^n} = n$

15) Solve eq-tion: $\left(a+(a+x)^{\frac{1}{2}}\right)^{\frac{1}{2}} = x$

16) Solve ineq-ty: $x\left(8\sqrt{1-x}+\sqrt{1+x}\right) < 11\sqrt{1+x}-16\sqrt{1-x}$

17) For which a does the eq[ ]tion: $4^x + 2 = a \cdot 2^x \sin ax$ have exactly one solution

18) Solve eq[ ]tion $\left(1 - \frac{1}{8}\cos^2 x\right)^8 = 2\sin^2 x$

19) Solve $\sin^7 x + \sin^{-3} x = \cos^7 x + \cos^{-3} x$

20) Solve $2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{\sqrt{2}}{2}}$

21) Find r of the inscribed circle in a triangle, knowing r of the circles, inscribed in the angles of the $\triangle$ and touching the inscribed circ. externally.

22) In a $\triangle$ a - r. of the circumscribed circ., b - of the inscribed circ., c - distance between their centers. Prove, that $c^2 = a^2 - 2ab$.

23) Prove, that S of an inscribed and circumscribed 4gon is equal to $\sqrt{}$ of the product of the sides.

24) In a circ. chords are drawn: BC (A midpoint), DP and EM, passing through A, D and E - on $\overset{\smile}{BC}$, EP and DM intersecting BC at pt.s K and M. Prove: $AK = KM$.

25) If all bisectors of a triangle < 1, its $S < \frac{\sqrt{3}}{3}$

26) If all bisectors of a $\triangle$ are equal, it is equilateral

27) Among all quadrilaterals with given sides find the one with the largest area.

28) Diagonals divide a 4-gon into 4 $\triangle$ of equal perimeter $\Rightarrow$ it is a rhombus.

29) Find the l.o.p. [locus of points], the sum of distances from which to 2 given lines is constant.

30) On a plane colored in 3 colors, there exist 2 points at a distance of 1 of the same color.

31) Using a compass construct the vertices of some square.

32) $\forall\, 3$ of k convex sets intersect, then all k - sets intersect. What can be said if their number is ∞, or not all are convex.

and in space (faces of a tetrahedron

33) Using only a straightedge, drop a $\perp$ from a given point to a given diameter of a given circle. (all cases)

34) Using a compass, divide a given line segment into k equal parts.

35) Construct a square given 4 points on its sides.

36) Construct a 4-gon given 4:

a) angles and diagonals,

b) sides and angle between diagonals,

c) sides and midline.

37) Cut off from a given angle via a line passing through a given point, a $\triangle$ of

a) given p;

b) minimal p.

38) Construct a network of roads of minimal length connecting

a) 3

b) 4

villages.

48) Prove the equivalence of the properties of a tetrahedron:

a) all altitudes inters. at one point;

b) one of the altitudes falls into the orthocenter of a face.

49) One tetrahedron is inside another. Is it true that for the 2nd tetr.

a) sum of lengths of edges

b) sum of S of faces

is larger?

50) All faces of a polyhedron - $\triangle$ Prove, that there exists an edge such that all 4 angles adjacent to it are acute.

51) A sphere is tangent to all sides of a spatial 4-gon. Prove, that all points of tangency lie in 1 plane.

52) Prove, that at least 1 of the perpendiculars dropped from a point inside a convex polyhedron onto its faces will fall onto the face, and not onto its extension.

54) Prove, that the sum of all dihedral angles of a tetrahedron is greater than 2π and $< 3\pi$. and can take any value within these limits.

55) $\log_2 3 \vee \log_3 5$

56) Rationalize the denominator of $\dfrac{1}{\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}}$

58) $\exists$ an α and rat. p and T, such that

$$\cos^2 \alpha + p\cos\alpha + T \,=\, 0 \,=\, \cos^2 2\alpha + p\cos 2\alpha + T,$$

and $\cos\alpha$ and $\cos 2\alpha$ - irrational.

62) $3^{\sqrt{2}} \vee 2^{\sqrt{5}}$

63) $\dfrac{8\pi}{27} \vee \sin\dfrac{8}{7}$

66) Prove, that $\sin 10°$ - irrat.

67) Construct a segment $\left(a^{\frac{1}{4}} + b^{\frac{1}{4}}\right)^4$.

68) Find $\operatorname{tg}\dfrac{\pi}{7}\operatorname{tg}\dfrac{2\pi}{7}\operatorname{tg}\dfrac{3\pi}{7}$